UMEÅ UNIVERSITY

Symmetries of Differential Equations

(Symmetrier för differentialekvationer)

Credit: 7.5 ECTS

Course coordinator:
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Course Period:
February – April 2026

Main field of study and progress level:
Mathematics, PhD

Prerequisites:
Students should have a basic knowledge of linear algebra, multivariable calculus, and differential geometry. In particular, students are expected to be familiar with the theory of functions and vector fields on manifolds, as well as the foundations of the theory of Lie groups.

Objective
The course will provide an introduction to the geometrical foundations of Lie point transformations and symmetries of ordinary and partial differential equations (ODEs and PDEs respectively), as well as a solid foundation for analysing and classifying differential equations with respect to their symmetries.

Contents:
Symmetries of a differential equation are transformations that map solutions to other solutions. Of particular importance is the case of continuous groups of symmetries, which provided the original motivation for the development of the theory of Lie groups. The analysis of symmetry groups of differential equations, and conversely the construction of differential equations which are manifestly invariant under some symmetry group, are important problems in mathematics and modern mathematical physics. Recently, symmetries have also been introduced in machine learning, in the field of geometric deep learning, where they encode properties of, e.g., neural networks and neural differential equations. To provide sufficient knowledge of the foundations of symmetries of differential equations for further study of both classical and more recent applications this course will cover:

- Point transformations and generating vector fields
- Prolongations of transformations and vector fields
- Symmetries of differential equations
- Differential invariants
- Characterisation of invariant differential equations
- Integration of ordinary differential equations
- Group invariant solutions

Form of instruction:
The teaching consists of lectures and self-study of the course books. The primary reading material

for the course is chapters 2-3 in [1], and the background material on differential geometry and Lie groups in chapter 1 as needed. The book [2] provides a complementary presentation of the material, while [3] dispenses with the geometric perspective and focuses primarily on the integration of ODEs using their symmetries.

Examination:
The examination consists of hand-in problems, to which complete written solutions are to be submitted to the course coordinator.

Literature:
The primary course literature is:

[1] Olver, P., *Applications of Lie Groups to Differential Equations*, Springer-Verlag, 1998.

Supplemental reading is:

[2] Olver, P., *Equivalence, Invariants and Symmetry*, Cambridge University Press, 2008
[3] Hydon, P., *Symmetry Methods for Differential Equations*, Cambridge University Press, 2000